An element-by-element algebraic multilevel block-ilu preconditioner

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Iterative solution methods are commonly employed to solve large sparse linear systems $A\mathbf{x} = \mathbf{b}$. Such systems arise, for example, from the finite element discretization of partial differential equations. These systems also have an underlying element structure. The system matrix can thus be expressed as an assembly of finite elements: $A := \sum_{e \in E} A^{[e]}$. The performance of iterative methods strongly depends on the choice of the preconditioner, however. It is well-known that an effective preconditioner should cluster the eigenvalues of the preconditioned system near unity.

A class of purely algebraic preconditioners, known to be effective in practice, are the algebraic multivel preconditioners. Algebraic multilevel preconditioners are based on the geometric multigrid method. Unlike classic multigrid methods, they are not restricted to simple geometries. They can, in principle, be applied to any sparse matrix. However, dense BLAS3 codes, generally, can not be applied during the construction or application of the preconditioner. Furthermore, many of these preconditioners are hard to parallelize.

Another class of preconditioners are the element-by-element preconditioners. They are renowned for their massive degree of inherent parallelism. Unfortunately, many of these preconditioners were derived in the context of some specific partial differential equations. Their performance on general sparse matrices is unproven, at best.

The incomplete LU factorization preconditioners, form another class. They are generally known for their applicability to and relative performance on a wide range of problems. Their theoretic justification is, however, only established for a limited class of matrices. Parallelization of ILU-type preconditioners is hard. Only a limited degree of parallelism can be extracted by techniques such as wavefront scheduling.

In this work, we aim to combine the strengths of the aforementioned approaches. We consider a new element-by-element algebraic multilevel block-ILU preconditioner for linear systems with an underlying element structure. The multilevel block-ILU factorization is formed in an element-by-element fashion. The resulting preconditioner has a high degree of inherent parallelism, uses BLAS3 routines during construction and application and inherits the theoretical justification from algebraic multilevel methods (under limiting assumptions). The resulting algorithm also resembles the multifrontal method well. Our numerical experiments demonstrate the effectiveness of our no-fill preconditioner relative to ILU(0) on a number of problems. A variant which allows fill-in, based on symbolic information only, is also considered.

Keywords: Numerical linear algebra, Partial differential equations, Element-By-Element, Block, ILU, Multilevel, Preconditioner.